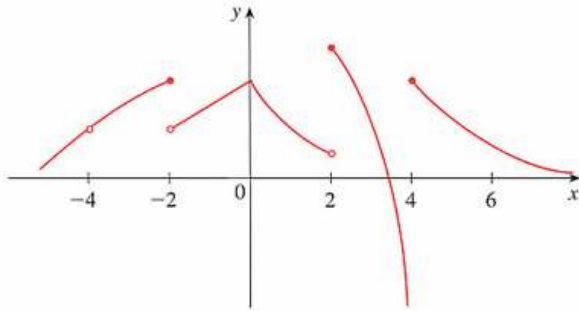
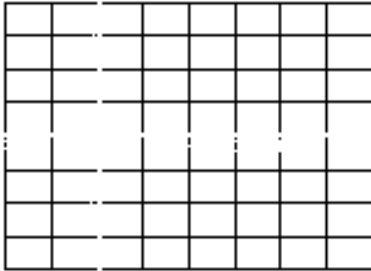


Limits and Continuity

1. (a) From the graph of f , state the numbers at which f is discontinuous and explain why.
 (b) For each of the numbers stated in part (a), determine whether f is continuous from the right, from the left, or neither.

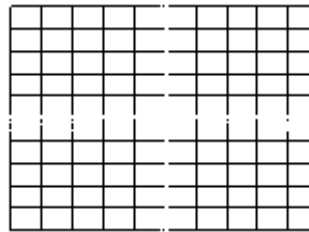


2. Sketch the graph of a function that has a jump discontinuity at $x = 2$ and a removable discontinuity at $x = 4$, but is continuous elsewhere.

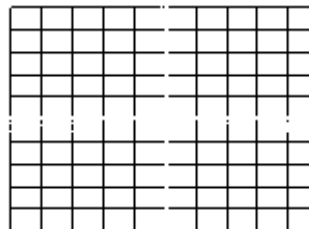


- 3-4: Find the numbers at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither? Sketch the graph of f .

$$3. f(x) = \begin{cases} 1+x^2 & \text{if } x \leq 0 \\ 2-x & \text{if } 0 < x \leq 2 \\ (x-2)^2 & \text{if } x > 2 \end{cases}$$



$$4. f(x) = \begin{cases} x+1 & \text{if } x \leq 1 \\ 1/x & \text{if } 1 < x < 3 \\ \sqrt{x-3} & \text{if } x \geq 3 \end{cases}$$



5. Identify each of the following functions as continuous or not continuous on $(-\infty, \infty)$.

a) $y = |x| - 4$

e) $y = x^{2/3}$

b) $y = |x - 4|$

f) $y = x^{3/2}$

c) $y = \sqrt{x^2 + 9}$

g) $y = \frac{x}{x^2 + 4}$

d) $y = \sqrt{x^2 - 9}$

h) $y = \frac{x}{(x+4)^2}$

6-8: Find the value of k so that the function will be continuous.

6. $f(x) = \begin{cases} x^2 - 4 & x \leq 3 \\ 2x + k & x > 3 \end{cases}$

7. $f(x) = \begin{cases} x^2 + 8x + 15 & x \neq -3 \\ k & x = -3 \end{cases}$

8. $\begin{cases} f(x) = \frac{4x^2 - 4x}{x - 1} & \text{if } x \neq 1 \\ f(1) = -2k \end{cases}$

9. $f(x) = \begin{cases} x - 3 & x < 1 \\ 5 & x = 1 \\ x^2 + 1 & x > 1 \end{cases}$

10. $f(x) = \begin{cases} x^2 - 3x + 1 & x < -2 \\ 11 & x = -2 \\ -4x^2 + 3 & x > -2 \end{cases}$

Find the following:

a) $\lim_{x \rightarrow 1^+} f(x)$

Find the following:

a) $\lim_{x \rightarrow -2^+} f(x)$

b) $\lim_{x \rightarrow 1^-} f(x)$

b) $\lim_{x \rightarrow -2^-} f(x)$

c) $\lim_{x \rightarrow 1} f(x)$

c) $\lim_{x \rightarrow -2} f(x)$

d) $f(1)$

d) $f(-2)$

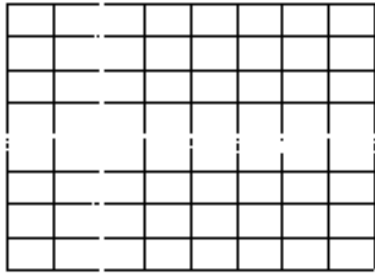
e) Is $f(x)$ continuous at $x = 1$?

e) Is $f(x)$ continuous at $x = -2$?

Limits and Continuity: KEY

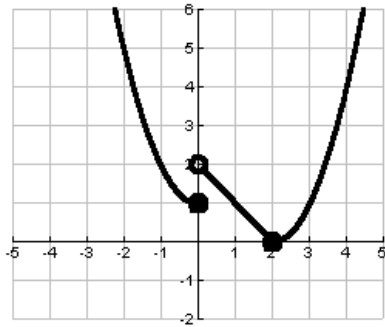
1. (a) The following are the numbers at which f is discontinuous and the type of discontinuity at that number: -4 (removable), -2 (jump), 2 (jump), 4 (infinite)

(b) f is continuous from the left at -2 since $\lim_{x \rightarrow -2^-} f(x) = f(-2)$. f is continuous from the right at 2 and 4 since $\lim_{x \rightarrow 2^+} f(x) = f(2)$ and $\lim_{x \rightarrow 4^+} f(x) = f(4)$. It is continuous from neither side at -4 since $f(-4)$ is undefined.

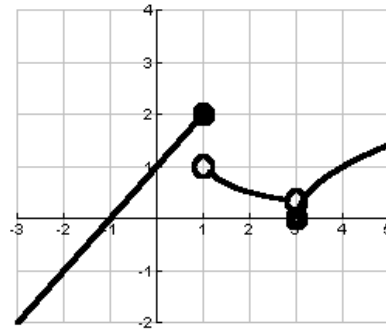


2.

3. f is discontinuous at 0.
 f is continuous from the left at 0.
 F is continuous at 2.



4. f is discontinuous at 1.
 f is continuous from the left at 1.
 f is discontinuous at 3.
 F is continuous from the right at 3.



- 5. a) Continuous
- b) Continuous
- c) Continuous
- d) Not Continuous
- e) Continuous
- f) Not Continuous
- g) Continuous
- h) Not Continuous

- 6. $k = -1$
- 7. $k = 2$
- 8. $k = -2$

- 9. a) 2
- b) -2
- c) DNE
- d) 5
- e) No

- 10. a) -13
- b) 11
- c) DNE
- d) 11
- e) No