Chapter 3 Notes and Examples

3.1: Derivatives of Polynomials and Exponential Functions

Constant functions

To derive constant functions all you must know is that they wind up equaling zero: $\frac{d}{dx}(c) = 0$; in other words $f(x) = 21 \longrightarrow f'(x) = 0$ always.

Power Functions

To derive power functions you take the degree of the power and multiply that number by the constant. You then subtract one from the degree of the power. This is what it looks like: $f(x) = ax^n \longrightarrow f'(x) = n \bullet ax^{n-1}$

For example: $f(x) = x^3 \longrightarrow f'(x) = 3x^2$.

Constant Multiple Rule

You can essentially ignore, or remove the constants:

$\frac{d}{dx}\left(cf(x)\right) \to c\frac{d}{dx}f(x)$

Sum Rule

If you have two terms being added together, you are allowed to derive each term individually: $\frac{d}{dx}\left(f(x) + g(x)\right) \longrightarrow \left(f'(x) + g'(x)\right)$

Difference Rule

If you have two terms being subtracted together, you are allowed to derive each term individually: $\frac{d}{dx}\left(f(x) - g(x)\right) \longrightarrow \left(f'(x) - g'(x)\right)$

Derivative of the Natural Exponential function

The derivative of e to anything is e to that anything times that anything prime.

$\frac{d}{dx}e^x \to e^x$ if you have e to anything other than x it looks like this: $\frac{d}{dx}e^{2x} \to 2 \bullet e^{2x}$

3.2: The Product and Quotient Rules

Product rule:

If two terms are multiplied together you cannot simply take the derivative of both; it doesn't work.

$$\frac{d}{dx}\big(f(x)g(x)\big) \neq f'(x) \bullet g'(x)$$

Instead it is necessary to use the Product Rule: leave the first and take the derivative of the second, plus derivative of the first time second left alone. It should look like this:

$$\frac{d}{dx}\big(f(x)g(x)\big) = f(x) \bullet g'(x) + f'(x) \bullet g(x)$$

Quotient rule:
The same principal holds true for division. You cannot take the derivative of both terms and have it work.

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) \neq \frac{f'(x)}{g'(x)}$$

There is a catchy tune to remember how to do derive this particular type of problem (it's the Quotient Rule song, yeah for Groller!) "low D hi minus hi D low all over the square of what's below" in other words:

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{lowDhi - hiDlow}{below^2}$$

or in formal notation:

$$:\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x) \bullet f'(x) - f(x) \bullet g'(x)}{g(x)^2}$$

3.3: Rates of Change in the Natural and Social Sciences

If a car (or particle) is traveling down a highway (or the x-axis), then, unless otherwise stated:

$f(x) =$ the position of the car or particle
$f'(x) =$ the velocity of the car or particle
$f''(x) =$ the acceleration of the car or particle
$f'''(x) =$ the jerk, or rate of change of the acceleration of the car or particle

---

3.4: Derivatives of Trigonometric Functions

Trigonometric Derivatives

Sin(x)	Cos(x)
Cos(x)	-Sin(x)
-Sin(x)	-Cos(x)
-Cos(x)	Sin(x)
Sec(x)	Sec(x)Tan(x)
Csc(x)	-Csc(x)Cot(x)
Tan(x)	$Sec^2(x)$
Cot(x)	$-Csc^2(x)$

Trigonometric Derivatives

$Sin^{-1}(x)$	$\frac{1}{\sqrt{1-x^2}}$
$Cos^{-1}(x)$	$-\frac{1}{\sqrt{1-x^2}}$
$Sec^{-1}(x)$	$\frac{1}{x\sqrt{1-x^2}}$
$Csc^{-1}(x)$	$-\frac{1}{x\sqrt{1-x^2}}$
$Tan^{-1}(x)$	$\frac{1}{1+x^2}$
$Cot^{-1}(x)$	$-\frac{1}{1+x^2}$

3.5: The Chain Rule

The **Chain Rule** is used to differentiate a composite function (a function within a function), such as $\sqrt{(x^3+4)}$. The way to differentiate F(x), if F(x) is defined as f(g(x)) is:

$$F'(x) = f'(g(x)) \bullet (g'(x))$$

For example, if F(x) is defined as $\sqrt{(x^3+4)}$, than F'(x) = $\frac{1}{2}(x^3+4)^{-\frac{1}{2}} \bullet 3x^2$

3.6: Implicit Differentiation

Since not all functions are easily defined in terms of one variable, such as $x^3 + y^3 =$ 6xy, it is necessary to use **Implicit Differentiation** to derive in terms of two variables. When implicitly differentiating, you must multiply the Leibniz notation for the derivative by all variables other than the one which you are deriving in terms of. For example:

$$F \Rightarrow\ x^3 + y^3 = 6xy$$

$$F' \Rightarrow\ 3x^2 + 3y^2\left(\frac{dy}{dx}\right) = 6x\left(\frac{dy}{dx}\right) + y(6)$$

$$F' \Rightarrow\ 3x^2 - 6y = 6x\left(\frac{dy}{dx}\right) - 3y^2\left(\frac{dy}{dx}\right)$$

$$F' \Rightarrow\ 3x^2 - 6y = (6x - 3y^2)\left(\frac{dy}{dx}\right)$$

$$F' \Rightarrow\ \left(\frac{dy}{dx}\right) = \frac{3x^2-6y}{6x-3y^2}$$

3.7: Higher Derivatives

Any time you derive beyond the first derivative (f'), it is known as a **Higher Derivative**. The process is the same, you simply have to derive f' again to get f'', and again to get f''', and so on. The notation for Higher Derivatives is as follows:

y', y'', y''', y^{iv}, y^{v} and so on;

or: $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}$ and so on.

3.8: Logarithmic Derivatives

Logarithmic Derivatives

a^x	$a^x \bullet \ln(a) \bullet x'$
$\log_a x$	$\frac{1}{x \bullet \ln(a)} \bullet x'$
$\ln(f(x))$	$\frac{f'(x)}{f(x)}$
$\ln(x)$	$\frac{1}{x \bullet \ln(e)} \bullet x'$ or $\frac{1}{x}$

3.10: Related Rates

First off, **Related Rates** are applications of implicit differentiation. And as the name implies, you must relate the rates within the problem. For instance with a falling latter, The distances of the horizontal and vertical components away from the floor and the wall respectively can be related through using Pythagorean Theorem.

Sample Problem: A ladder is against a wall with the height of the ladder being 10 meters. If at one particular instance the horizontal distance from the bottom of the ladder to the wall is 8 meters and the vertical distance from the top of the ladder to the floor is 6 meters. If the rate at which the vertical distance is decreasing is 4 meters per second how fast is the horizontal distance from the bottom of the ladder to the wall increasing?

Solution: First take the Pythagorean Theorem and implicitly differentiate.

- $z^2 = x^2 + y^2$ after implicit differentiation $2z\left(\frac{dz}{dt}\right) = 2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right)$
- Then plug in values from problem $2(10)(0) = 2(8)\left(\frac{dx}{dt}\right) + 2(6)(-4)$
- Then solve for dx/dt which is equal to 3 meters per second

Remember to differentiate before plugging in values. Also indicate whether the rate is positive or negative to show increasing or decreasing value.

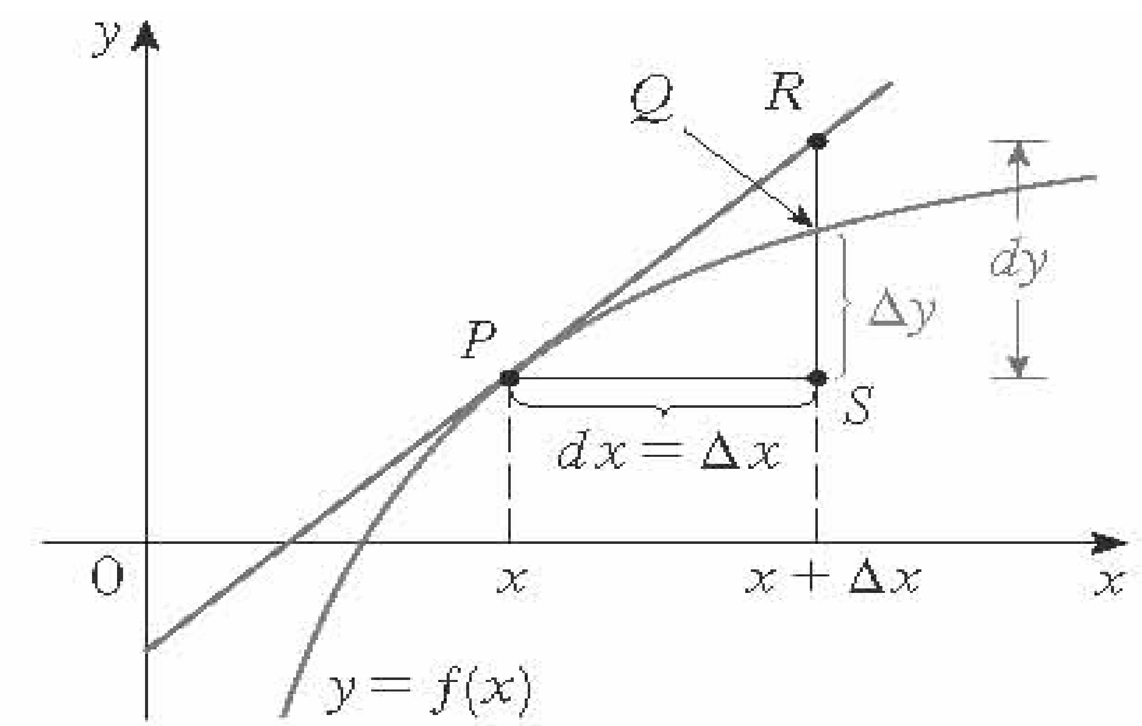
3.11: Linear Approximations and Differentials

Linearization:
Linear Approximation or tangent line approximation is used to approximate the future behavior or curve of $y = f(x)$ based on a point f(a) is commonly known as **Linearization** and is written:

$$L(x) = f(a) + f'(a)(x-a)$$

Differentials:
Differentials are used to better approximate the difference between the real function and its linear tangent.

$$\frac{dy}{dx} = f'(x) \longrightarrow dx \bullet \frac{dy}{dx} = f'(x) \bullet dx \longrightarrow dy = f'(x) \bullet dx = f'(a)(x-a)$$



$dy =$ change of linear approximation function from $a \longrightarrow b$ on the y-axis

$\Delta y =$ change of original function from point $a \longrightarrow b$ on the y-axis

$\Delta x = dx =$ change from $a \longrightarrow b$ on the x-axis